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A. L. Foote furnished a neat algebraic solution; *G. B. M. Zerr*, *P. S. Berg*, *Cooper D. Schmitt* solved the problem by the calculus, and *C. D. M. Showalter* gave a good geometrical solution. Space forbids further consideration of this problem.

38. Proposed by LEONARD E. DICKSON, M. A., Fellow in Mathematics, University of Chicago.

Give a *strictly geometric* proof of my fundamental theorem of the Inscription of Regular Polygons, viz: Suppose a circle of unit radius divided at the points $A, A_1, A_2, A_3, \dots\dots A_p, \dots$ into $2p+1$ equal parts and the diameter AO drawn. Then, if the chords $OA_1, OA_2, \dots\dots OA_p$ be drawn, we have $OA_1-OA_2+OA_3-OA_4+OA_5-\dots\dots\pm OA_p=1$.

Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

For plainness we will solve this rpoblem in full for the 13-gon. A general solution is as easy, but not as clearly understood.

Let OA_6, OA_5, OA_4, &c. $=a_1, a_2, a_3$, &c.

$$A_4A_5=A_5A_6=A_6A_7=\&c=c,\quad A_4A_6=A_5A_7=A_6A_8=\&c.=d.$$

Now by Ptolemy's Theorem:—The rectangle contained by the diagonals of a quadrilateral inscribed in a circle &c., we easily get the following relations:

$$\begin{array}{l|l|l}
c(a_1+a_3)=da_2 & c(a_6+a_8\ )=da_7 & c(a_{10}+a_{12})=da_{11} \\
c(a_2+a_4)=da_3 & c(a_7+a_9\ )=da_8 & c(a_{11}+a_{13})=da_{12} \\
c(a_3+a_5)=da_4 & c(a_8+a_{10})=da_9 & c(a_{12}-a_1\ )=da_{13} \\
c(a_4+a_6)=da_5 & c(a_9+a_{11})=da_{10} & c(a_2\ -a_{13})=da_1 \\
c(a_5+a_7)=da_6 & &
\end{array}$$

Hence $d\{(a_1+a_3+a_5+a_7+a_9+a_{11}+a_{13})-(a_2+a_4+a_6+a_8+a_{10}+a_{12})\}$
$=c\{(a_2-a_{13})+(a_2+a_4)+(a_4+a_6)+(a_6+a_8)+(a_8+a_{10})+(a_{10}+a_{12})+(a_{12}-a_1)$
$-(a_1+a_3)-(a_3+a_5)-(a_5+a_7)-(a_7+a_9)-(a_9+a_{11})-(a_{11}+a_{13})\}$.

$\therefore\quad (d+2c)\{(a_2+a_4+a_6+a_8+a_{10}+a_{12})$
$-(a_1+a_3+a_5+a_7+a_9+a_{11}+a_{13})\}=0$.

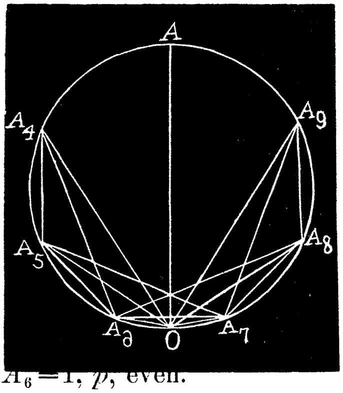
$\therefore\quad a_2+a_4+a_6+a_8+a_{10}+a_{12}$
$=a_1+a_3+a_5+a_7+a_9+a_{11}+a_{13}$.

Generally $a_2+a_4+a_6+\dots\dots+a_{2p}$
$=a_1+a_3+a_5+\dots\dots+a_{2p+1}$.

In the above, O can be any point between OA_p and OA_{p+1}.

In the problem $OA=a_7=2$, $a_6=a_8$, $a_4=a_6$, $a_2=a_4$, $a_1=a_{13}$, $a_3=a_{11}$, $a_5=a_9$.

$\therefore\quad a_2+a_4+a_6=a_1+a_3+a_5+1$, but $a_6=OA_1$, $a_5=OA_2$, $a_4=OA_3$, $a_3=OA_4$,
$a_2=OA_5$, $a_1=OA_6$.



$\therefore\quad OA_1-OA_2+OA_3-OA_4+OA_5$
$-OA_6=1$, p, even.

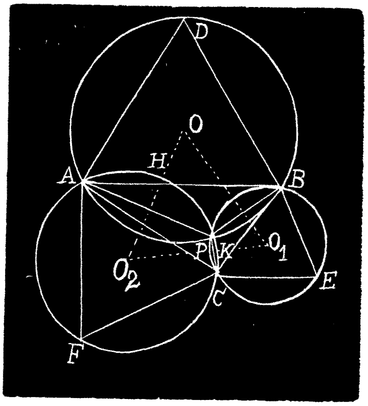
For 15-gon. $OA_1-OA_2+OA_3-OA_4+OA_5-OA_6+OA_7=1$, p, odd.

$\therefore\quad OA_1-OA_2+OA_3-OA_4+OA_5-\dots\dots\pm OA_p=1$, as p is odd or even.

39. Proposed by J. K. ELLWOOD, Principal of Colfax Schools, Pittsburg, Pennsylvania.

If on the three sides of any plane triangle equilateral triangles be described, the lines joining the centres of these equilateral triangles form an equilateral triangle.

I. Solution by F. E. MILLER, Ph. D., Professor of Mathematics, Otterbein University, Westerville, Ohio; Professor J. F. W. SCHEFFER, A. M., Hagerstown, Maryland; JOHN T. FAIRCHILD, Ada, Ohio; J. C. CORBIN, Pine Bluff, Arkansas; and the PROPOSER.



Let ABC be any plane $\triangle$, O, O_1, and O_2 the centres of the equilateral $\triangle$'s constructed. About these $\triangle$'s pass circumferences. They will intersect in a point, P.

Let P be the intersection of the 2 circles, AFC and CEB. Join AP, CP and BP. Since $APCF$ is inscribed, $\angle F + \angle APC = 180°$. But $\angle F = 60°$. $\therefore$ $\angle APC = 120°$. Similarly, $\angle CPB = 120°$. $\therefore$ $\angle APB = 120°$; and $\angle APB + \angle D = 180°$. $\therefore$ $APBD$ is inscribed, and P is in the circumference of DAB. (Q. E. D.)

Lines that join the centres of intersecting circles bisect the common chords and the intercepted arcs. $\therefore$ arc $HP = \frac{1}{2}$ arc AP; and arc $PK = \frac{1}{2}$ arc PC. $\therefore$ arc $HPK = \frac{1}{2}$ arc APC. But arc APC measures the angle $F = 60°$ at circumference; therefore its half HK measures an equal angle at the centre.

$\therefore$ $\angle O_2 = \angle F = 60°$. Similarly, $\angle O_1$ may be shown $= \angle E$, and $\angle O = \angle D$. But equiangular $\triangle$'s are also equilateral. $\therefore$ OO_1O_2 is equilateral.
Q. E. D.

II. Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia, and Professor J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Let O, O_1, O_2 (Fig. in Solution I.) be the centres of the equilateral triangles on the sides BA, BC, AC respectively, of the triangle ABC. Let $\triangle$ = area ABC.

Then $\angle OAB = \angle OBA = \angle O_1BC = \angle O_1CB = \angle O_2CA = \angle O_2AC = 30°$.

$$OB = OA = \frac{c}{\sqrt{3}},\quad O_1B = O_1C = \frac{b}{\sqrt{3}},\quad O_2C = O_2A = \frac{a}{\sqrt{3}}.$$

$$\therefore\quad OO_1 = \sqrt{\frac{c^2}{3} + \frac{b^2}{3} - \frac{2bc}{3}\cos.(60° + A)}$$

$$= \sqrt{\frac{c^2}{3} + \frac{b^2}{3} - \frac{1}{3}bc\cos. A + \frac{\sqrt{3}}{3}bc\sin A}$$

$$= \sqrt{\frac{1}{6}(a^2 + b^2 + c^2 + \sqrt{3}\triangle)} = O_1O_2 = O_2O$$

$\therefore$ OO_1O_2 is equilateral.

Excellent solutions of this problem were also received from *Professors F. P. Matz* and *G. I. Hopkins.*